Study some examples of Problem-based Learning applied to Mathematics in the Engineering Bachelor.

Juan José Reyes Salgado[1]

Resumen

En este documento se realizó el estudio de dos problemas, aplicando el aprendizaje basado en problemas, para cursos de matemáticos en la Licenciatura de Ingeniería. La implementación de esta técnica tiende a ser tema de debate entre los docentes, debido al poco entendimiento de esta técnica. La intención de este trabajo es mostrar como se aplicaron estas actividades de aprendizaje basado en problemas y la experiencia que nos comparten estos estudios, para que nuestra comunidad de docentes también sea capaz de diseñar de manera exitosa este tipo de actividades del aprendizaje activo.

Palabras Clave: Aprendizaje basado en Problemas, Aprendizaje Significativo, Aprendizaje activo.

Abstract

In this document, two problems were studied, applying problem-based learning, for mathematics courses in the Bachelor of Engineering. The implementation of this technique tends to be the subject of debate among teachers due to the little understanding of this technique. This work intends to show how these problem-based learning activities were applied and the experience that these studies share with us so that our community of teachers can also successfully design this type of active learning activity.

Keywords: Problem-based Learning, Significative Learning, Active Learning

1 Introduction

Traditional Education methodology is based on teacher exposition or reading a book. In other words, it promotes passive learning under which the student doesn't interact with the knowledge and only covers the course guide by providing minimum significant learning, even nothing. Active learning is an alternative to this kind of problem in teaching.

[1] Engineering Bachelor of Universidad Popular Autónoma del Estado de Puebla, email: juanjose.reyes@upaep.mx

Active learning is an approach in which students actively, rather than passively, receive information. It can include various activities and techniques such as discussions, group work, problem-solving, and hands-on activities [1].

John Dewey promoted experiential learning at the beginning of the 20th century and stressed the importance of linking learning to individual experience [2]. In the 1960s, Jean Piaget developed the constructivist learning theory, emphasizing that learners construct knowledge actively rather than passively receive it [3]. Later, in the 1980s, David Kolb proposed his experiential learning model that focused on reflection and experience-based learning [4].

Nowadays, active learning has continued to gain popularity, especially with the advancement of digital technology that enables more interactive and immersive learning experiences. Many universities and schools worldwide have incorporated active learning techniques into their curricula. Studies have shown that students who engage in active learning have better academic outcomes than those who do not. However, some students and teachers need help to adopt these techniques [5].

Problem-based learning is one of the most used Active Learning Techniques. This student-centered methodology emphasizes critical thinking, reflection, and teamwork[6]. It is an approach that enables learners to identify gaps in their knowledge, which motivates them to seek new information to fill them actively. It focuses on solving real-world problems or scenarios through open-ended questions or inquiries rather than memorizing information for exams.

The pioneer university in applying the Problem-based learning technique was Alborg, Denmark. Other universities and institutes developed several studies between 1992 and 2013, and nearly 15 countries published over 2551 papers on this topic. United States, United Kingdom, Canadian, Australia, and Netherlands represent 64% of the published papers [7].

- The Problem-based Learning process involves the following steps:
- Finding a problem.
- Presented to students who worked on the problem in small groups. Each student assumes a role within the group, which may be formal or informal.
- The students develop hypotheses and present evidence to support their ideas. The process allows learners to construct their learning, develop their research and reflection skills, and build upon their existing knowledge.
- The tutor, the learning process facilitator, supports and guides the learners as they work through the material. The tutor aims to build the learners' confidence when addressing problems and to help them expand their understanding of the topic. Problem-based learning encourages critical thinking, enhances group collaboration and communication, and promotes ongoing learning and growth within a team environment.

It represents a shift from traditional teaching, often lecture-based, to a more interactive and engaging way of learning. Despite requiring more preparation time and resources to support small-group learning, studies have shown that problem-based learning can lead to better academic outcomes and a more profound understanding among students.

The present study analyzes successful examples of Problem-based Learning applied to mathematics in Engineering Bachelor.

2 Study cases of Problem-based Learning

The problem-based learning must be applied considering students' desired development of knowledge and skills. The pedagogical challenge outline must be analyzed in the following [7]:

- Pedagogical Task.
- Curricular Contents (to Apply).
- Scientific Knowledge (to Discover).
- Skills (to Develop).

This study presents two successful cases of applying problem-based learning focused on mathematics for bachelor engineering students. The first case shows how problem-based learning is used to teach antiderivatives and compute areas in a subject related to **Calculus** [7]. The second case is associated with First Order Differential Equation Applications in the course of **Differential Equations**[8].

2.1. *First Case: Problem-based Learning Applied to Calculus.*

County and year: Portugal 2020

University: Open University of Portugal.

Background: Rézio et al. (2022) selected a group of first-grade students of Bachelor Engineering to apply the technique, and their population was 11 students of Aeronautic Engineering, one student of Material Science Engineering, and six students of Mechanical Engineering. The work team is allowed a maximum of 4 members.

Problem Description: As seen in Fig. 1, aircraft geometry objects proposed by Rézio et al. (2022) were modeled in 2D by each team.

Review: Intructors provide a 120-min class in which students paid attention to the part that had been assigned to them. Later, they considered mathematical modeling to solve the problem. Specifically, students selected the piece's position in a 2D Cartesian frame, realizing that the boundary lines of the pieces of paper corresponded to graphs of functions. Hence, they discovered the analytical expressions that best suited the previous figures, began using graphic software "Desmos" to reproduce the piece graphically, and cleared any misunderstandings with the teacher. Students worked independently for two weeks. Oral presentations and work discussions were held.

Fig. 1. Bidimensional Aircraft Geometry Objects proposed by Rézio et al. (2022)

The Pedagogical Challenge was designed and implemented by Rézio et al. (2022) in four stages:

1. Conception by the teacher.
2. Presentation of the proposal to the students and its resolution.
3. Student presentation to the group class and discussion.
4. Students' final reflections.

They exposed the following questions to the students:

1. How should the object be placed in a bi-dimensional frame?
2. What are the algebraic expressions of the functions involved (at the object's boundary)?

3. What is the object's surface area (in square centimeters)?
4. A variation on the proposed challenge.

The pedagogical challenge of the course of Calculus 1 is shown in Table 1. Describing the Pedagogical Task, Curricular Contents, Scientific Knowledge, and Skills students are expected to develop by applying Problem-based Learning.

Table 1. Pedagogical Challenge Outline of the First Case Studied [7].

Pedagogical Task	Curricular Contents (to Apply)	Scientific Knowledge (to Discover)	Skills (to Develop)
Pedagogical Challenge	Graphical representation of a function based on its algebraic expression.	The decision to place the piece in a bi-dimensional reference is critical for reducing/increasing the difficulty of algebraic function determination.	Retrieve the piece sketch, and use graphical software (Desmos).
In Search of a Bi-Dimensional Geometric Model	Use integral calculus to calculate the area of a surface.	Option to model a surface's boundary using algebraic expressions of functions. It is worth noting that some quadratic functions have a graphical representation similar to a semi-circumference. Modeling involves some degree of inaccuracy.	Extract the numerical calculations using integral calculus software (Symbolab).

Finally, their students presented the algebraic resolutions, images, and links to digital tools. Presentations lasted about 15 min and had no more than seven slides (forced condition). The displays explained how the problem was interpreted, how it was solved, how it was extended, and which digital tools were used [7].

2.2. *Second Case: Problem-based Learning Applied to Differential Equations.*

County and year: Mexico 2015.

University: Universidad Nacional Autónoma del México.

Background: The paper does not have any information about the population of the study group; Castaño Meneses only mentions that two work teams were formed.

Problem Description: Castaño Meneses (2015) showed his students a selected journal paper and gave them two tasks to discuss. On the one hand, there is the deduction of the linearized Poisson-Boltzmann equation for the case of zwitterions, and on the other is the solution of the equation in the cylindrical coordinates. The goal is to analyze the shielding parameter of a system polymeric.

Review: Both teams jointly initiate their respective task through a bibliographic search of some important events for developing polyelectrolyte theories, which the teacher had already specified[8].

For the deduction of the linearized Poisson-Boltzmann equation, the discussion of the task through the exposition by the Castaño Meneses of the field topics and the potential electrical, arriving until the exposition of the equation of Poisson.

From this point, the teams began brainstorming only with the member of their team. Once the topics of electric field and potential have been exposed, the students of one of the teams obtain an expression, in linear terms, for the approximation of the mean field of the charge distribution. Then, the teams had to express this equation in cylindrical coordinates.

Once both teams have completed the tasks and the respective conclusions have been made, Castaño Meneses (2015), as a seminar, develops the solution with the conditions at the border specified by the authors.

The pedagogical challenge of the course of Differential Equations is shown in Table 2. Describing the Pedagogical Task, Curricular Contents, Scientific Knowledge, and Skills students are expected to develop by applying Problem-based Learning.

Table 2. Pedagogical Challenge Outline of the Second Case Studied.

Pedagogical Task	Curricular Contents (to Apply)	Scientific Knowledge (to Discover)	Skills (to Develop)
Pedagogical Challenge	Analyze and apply the Poisson Equation.	Deduce the mean-field linear approximation of the charge distribution.	Deduce the Second Order Differential Equations from Potential Function.
Deduce the linearized Poisson-Boltzmann equation for the case of zwitterions.	Apply the methods to solve a Second Order Differential Equation.	Obtain the generalization for zwitterions of the Debye-Hückel equation. Modeling the infinite coaxial cylinder electrical charge.	Transform the functions and equations to cylindric coordinates.

3. Results of study cases of Problem-based Learning

The results obtained in each case are commented on below:

3.1. *Results of First Case: Problem-based Learning Applied to Calculus.*

Rézio et al. (2022) students' answers summarized the main ideas of the activity questions:

1. How should the object be placed in a bi-dimensional frame?
    a. Enable understanding of the values used in calculating algebraic expressions of boundary functions.
    b. The figure is divided in half.
    c. The OX axis divided the triangle's semi-circumference (figures identified as part of the object).
    d. The bi-dimensional object's longest side was parallel to one of the axes.

e. The parabola is identified as part of the boundary, with an upward concavity.

2. What are the algebraic expressions of the functions involved (at the object's boundary)?
    a. Some students traced the piece and then drew the axes on paper. Then, they attempted to deduce what kinds of functions were on the boundary and what algebraic expressions they contained.
    b. Others decided to use Desmos right away, introducing the functions that appeared to be appropriate and having the ability to visualize the graph. After that, they took the piece and attempted to superimpose it on the graph and see if they were equivalent.
    c. The significant difficulty identified at this challenging stage was some students' hesitation in choosing a semicircle or a parabola for some boundaries.
3. What is the object's surface area (in square centimeters)?
    a. Students understood the importance of decomposing the piece into different figures and deciding whether to add or subtract areas. As a result, some chose to define the integrals representing the various sub-areas and calculate them manually, or they also used Desmos.
4. A variation on the proposed challenge.
    a. One of the groups attempted to recreate accurate measurements by graphically recreating the approximate shape of a hang-glider using two parabolas and two triangles and calculating its approximate area in square meters.
    b. Another group investigated calculating a parabola arc's length and presented its mathematical formulation.

c. One group proposed using a GoogleMaps photo with an overhead view to calculate the capacity of a swimming pool, specifically the oceanic pool of Oeiras (known as a depth value).

The teacher who participated in this study was concerned with proposing a pedagogical activity to stimulate their curiosity and pro-activity. Essential skills that awaken reflective thinking in students include observation, communication, teamwork, judgment, and decision-making [9]. As a result, the Problem-based learning methodology was used to foster these abilities.

The teachers are no longer the only carriers of knowledge [10]; they also incorporate teaching initiatives, such as the ability to reflect on their practices, into their classes. The value of teachers and what they can do in the classroom should be acknowledged. According to the literature, the student's involvement in the process was an evident and anticipated aspect from their point of view. Their availability, whether on the part of the teachers or the students, was critical to the challenge's success.

3.2. *Results of Second Case: Problem-based Learning Applied to Differential Equations.*

Castaño Meneses (2015) engineering students well received the Problem-based Learning methodology. One point where all the students agreed was to change their role in the classroom, going from a passive attitude to an active attitude in which even the depth with which the topics are touched corresponds to them to establish. On the other hand, the teacher accepted both criticism and other possible solutions to the problems posed in the classroom so that the students participated more in the discussions.

The main problem the group faced was time, mainly because all the courses are still designed based on models of direct teaching and receptive learning, in addition to the fact that the

most used is the master lesson. For this reason, the group had to make an extra-class effort to meet the goals set.

From the author's experience, this type of methodology, where students are the center of the teaching-learning process, can be perfectly implemented in engineering careers since there is already a history of students who select the thesis as a degree work under a similar scheme. However, on many occasions, informal from the point of view of education, that is, it is enough that the student arrives at the result sought without evaluating how it is achieved or the added value that could be obtained during the development of his thesis work [8].

4. Discussion and Conclusions.

The two cases showed different styles of the application of Problem-based Learning in courses of mathematics in universities. The first case left the students to investigate and apply other software; also, they were free to solve the problem with any technique. In the second case, the teacher guided the activity, and the students decided on the depth of the knowledge.

It is worth recalling that teachers must be ready to apply any activity using the problem-based learning technique. So, they should know another point of view to solve the problem, and the student should decide the knowledge level he wishes to archive.

It is increasingly important for universities to add these active learning activities to their courses since the generations of students entering universities have more access to information but need to learn how to transfer it to applied and significant knowledge.

The application of these teaching techniques should not become torture for teachers. The activity can work optimally as long as a problem is raised that covers particular learning of the course and

skills that the student wants to acquire during the course. For this reason, it is recommended to use the tables of the pedagogical challenges, as seen in Tables 1 and 2, when selecting the problem.

The teacher must simplify the application of the activities using the problem-based learning technique considering the four steps shown in the Introduction on the processes involved in the method, as well as designing mechanisms for the evaluation of each of these parts to verify if students are acquiring the knowledge and skills that will lead them to solve the activity correctly.

The teacher must consider the intention of the students to acquire the knowledge of the course while applying the active learning technique; due to this, it is necessary to carry out a standard evaluation to ensure this appropriation and mastery of the skills and knowledge. The academic body of each course must develop these types of assessments.

In the university's mathematics area, the implementation of the problem-based learning technique is sought in the different courses that are taught. It is important to highlight that teaching in front of a group is still part of the teaching work; it is simply necessary to find the most appropriate balance for students to get the most out of classes.

Acknowledgments

Thanks to all the teaching community in the university's mathematics area with whom the interest and questions these active learning techniques bring have been shared.

This document does not present any conflict of interest.

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Caption List

Fig 1. Bidimensional Aircraft Geometry Objects proposed by Rézio et al.

Table 1. Pedagogical Challenge Outline of the First Case Studied.

Table 2. Pedagogical Challenge Outline of the Second Case Studied