Setting prices for second-hand video games

Fijación de precios de video juegos usados

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Abstract

In the transition to the purchase and sale of video games in its digital format, the second-hand market has grown significantly given the importance that the consumer provides to physical video games. Given the uncertainty that exists about the quality of a video game, videogame stores have become intermediaries that facilitate the purchase/sale and guarantee the quality of these. In this document, we analyse the interaction between a store and a video player, where each agent initially owns a video game used to determine the price of the object. The agents interact through a three-stage game with incomplete information where the store sets the purchase and sale prices of the used video games without knowing the valuation that the video player has on them. It is shown that there is a unique subgame perfect Nash equilibrium when the valuations are independent and follow a uniform continuous distribution. The comparative statics shows a positive relationship between the equilibrium prices and how the video player values the video games; Also, it is observed that, in equilibrium, the video player does not sell his good.

Keywords: Subgame perfect Nash equilibrium, second-hand markets, pricing.

Resumen

En la transición a la compra y venta de video juegos en su formato digital, el mercado de segunda mano ha crecido significativamente dada la importancia que el consumidor le da a los video juegos físicos. Dada la incertidumbre que existe sobre la calidad de un video juego, las tiendas de videojuegos se han convertido en intermediarios que facilitan la compra/venta y garantizan la calidad de estos. En este documento, se analiza interacción de una tienda y un video jugador, donde cada uno posee inicialmente un video juego usado, para determinar el precio del objeto. Los agentes interactúan a través de un juego de tres etapas con información incompleta donde la tienda fija los precios de compra y venta de los videojuegos usados sin conocer la valoración que el video jugador tiene sobre ellos. Se muestra que existe un único equilibrio de Nash perfecto en subjuegos cuando las valoraciones son independientes y siguen una distribución continua uniforme. La estática comparada muestra una relación positiva entre los precios de equilibrio y la forma en que el video jugador valora los videojuegos; además, se observa que, en equilibrio, el video jugador no vende su propio bien.

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Palabras Clave: Equilibrio perfecto en subjuegos, mercados de segunda mano, fijación de precios.

1 Introduction

The launch of a video game, or only game, characterises by setting a high introductory price and, in the majority of cases, a limited distribution that has motivated their consumer to look for better prices and a greater variety of titles in the second-hand market (Guiot and Roux, 2010). This is not a recent phenomenon due to the previous features have remained in the launching of the video game since their boom in the eighties, where the mass distribution of video games started for the general public (Milington, 2016).

Despite the growth of digital games sales in the last decade, the exchange of physical copies in second-hand markets still represents a significant interaction in the video games industry (Cox, 2017). In such markets, stores have market power to set prices which provides them significant net revenues by taking advantage of collectors' valuations. GameStop, an American retailer of video games, is a successful example in the purchase and sale of used video games; in its 2014 financial report, this company reported to its investors that 40% of its profits come from the sale of used video games, which represented a net profit of more than 1100 million dollars. The above has motivated other stores, previously focused on the sale of premiere video games, to enter the used video game market by buying the video games that their customers no longer wish to sell at a price they set themselves. For example, companies like Amazon and Wal-Mart entered this market in 2014 and 2015, respectively.[2]

[2] Recuperado de http://www.newyorker.com/business/currency/why-used-video-games-are-such-a-big-business

In the interaction between consumers and sellers, of used video games, it is not clear how sellers set prices of used games since listing prices of such games are different across sellers while consumers can sell their games at the same time when they buy a used game (Ishihara and Ching, 2019). In this paper, we analyse the setting of prices through the strategic interaction between a store and a video player; we assume that each agent has an initial endowment composed by an indivisible good, which is a used video game and an amount of money that agents use to pay the bill in the transactions.

Given the durability of video games, we assume that each agent has different valuations over the games that each of them owns. On the one hand, it is assumed that the video player values each asset in the market differently and privately. That is, the type of video player is his valuation vector. On the other hand, by simplicity, we consider that the store is willing to buy and sell the goods in the market, so its type is unique. The interaction between agents is described through a three-stage game with incomplete information. In the first stage, nature determines the type of video player, which is not observed by the store. During the second stage, the store sets the sale price of its good and the purchase price of the other agent's good. Finally, in the third and last stage, the video player observes the price vector that the store set in the previous stage and decides between the following four actions: 1. only buy the good from the store, 2. only sell his good, 3 Buy the good from the store and sell his good, and 4. Do not buy or sell. In other words, we model the interaction between the video player and the store as a three-stage game under agents can be sellers and buyers. So, we analyse the set of Subgame perfect Nash equilibria as the solution concept of the previous game. By following a backward induction process, we demonstrate the existence and uniqueness of the game's solution.

The study of second-hand markets has been performed from different approaches. One of them relates to durability because used goods compete with their new versions, even if the lasts have “new” features (Ishihara and Ching, 2019). Numerous articles study the price formation of durable goods through dynamic models and the impact of used products on the prices of new goods. Assuming a monopolistic market, Coase (1972) shows how the durability of goods reduces the market power that the monopolist possesses when he ignores the valuations of consumers in the market. Coase's result is consistent when there is a finite number of a consumer, as Bagnoli, Salant and Swierzbinski (1989) show, and when new consumers are allowed in each period, Sobel (1991). However, when consumer valuations are stochastic, the monopoly's market power increases. Also, Waldman (1996, 2003) shows that price discrimination among consumers can increase the benefits of a monopoly when there is an active market for second-hand goods. Besides, Morita and Waldman (2004) show that companies monopolise the markets for maintaining their products when new and used goods are imperfect substitutes. Thus, the literature analyses how the presence of a second-hand market impacts the demand and supply of new durable goods.

Thomas (2003) demonstrates that the demand for new goods increases in the absence of used goods, however, the rate of substitution of a new good for a used good is less than one when the latter is not fully utilised. Considering a dynamic model, Gowrisankaran and Rysman (2009) propose models on how the demand for durable goods changes when consumers can replace their products. Nair (2007), Dubè et al. (2010) and Liu (2010) provide empirical evidence on how the introductions of new video games impact the demand of used video games given the presence of price discrimination mechanisms in such market. Our paper contributes to this literature by analysing the price formation process of second-hand games.

The existence of incomplete information has also been studied to understand the behaviour of used goods markets. The paper of Akerlof (1970) on used cars is a classic article in this branch of literature; through the "model of lemons," Akerlof shows how the quality of cars decreases in the market because sellers have information that buyers do not have. In other words, Akerlof studies a second-hand market as a problem of adverse selection; under sellers have incentives to sell all cars with lousy quality. This approach has been addressed by other authors such as Rothschild and Stiglitz (1976), Bond (1982), van Cayseele (1993), and Hendel and Lizzeri (1999b) in the labour and financial markets where people misreport information to get a job or a credit.

In our work, we propose an analysis based on game theory due to the existence of private information since the store does not know video players' valuations; so, each agent wants to get the largest possible payoff at the end of the interaction. Our model differs from adverse selection models because we consider that the characteristics of goods are common knowledge. Therefore, our main contribution lies in analysing how the valuations of agents intervene in the formation of prices of second-hand video games. Also, we provide a closed-form for equilibrium prices when valuations are distributed uniformly, which allows us to perform comparative statics.

This article is structured as follows. In the second section, we present the model that describes the market of second-hand videogames; also, in this section, we describe the three-stage game through which the agents interact, the payoffs that agents get when the game finishes, as well as the concept of the solution used to solve the game. The third section focuses on the resolution of the game through the backward induction process; we compute the player’s decision rule at equilibrium, as wells as the equilibrium price vectors. When valuations are distributed independently and uniformly but not identical, there is a unique Subgame perfect Nash equilibrium. The fourth section shows the conclusions and possible extensions of our model.

2 The Model

By simplicity, we consider the second-hand market as a market with two agents, two video games, which are indivisible goods to be exchanged, and money. The latter is a fully divisible asset used by agents to pay for transactions; we use ω to represent a generic amount of money.

The set of agents is $J = \{s, g\}$ where s is the store and g denotes the video player (or player); we use a to indicate a generic agent in J. We consider that each agent initially owns a video game, which we denote by β_a. Thus, the set of indivisible goods in the market is $G = \{\beta_s, \beta_g\}$. Also, we assume that each agent has an initial amount of money ω_a, where ω_a is non-negative. To summarise the previous discussion, we assume that each agent is initially endowed with a basket e_a composed by an indivisible good and a positive amount of money; mathematically, we have that $e_a = (\beta_a, \omega_a) \in G \times R_+$.

Even though both agents initially own a good-money basket, we consider that agents are not homogeneous; that is to say, each agent is identified by a type that summarises how they value each good in the market. Concerning the store, we assume that s is willing to buy and sell any good in the market, i.e., the store is indifferent between all goods in G. Consequently, the store has a unique type. In opposition, the player has different valuations for each good in G; we denote by v_{ga} be the valuation of agent g of good β_a for all a$\in J$. So, the type of player g is the valuation's vector $v_g = (v_{gg}, v_{gs})$. We denote by V_g the set of all possible types of agent g, note that $V_g \subseteq \mathbb{R}^2$. We assume that the player's type is private information, i.e., the store does not know v_g, which is the realisation of the random vector $V_g = (V_{gg}, V_{gs})$.

The state of the market is the type vector $v = (v_a, v_g)$, where $V_g \times V_s$ is the set of all possible market states. Given that types are private information, note that each $v \in V_g \times V_s$ is the realisation of the random vector $V = (V_a, V_g)$. We consider that V follows a probability function $f: V_g \times V_s \to [0,1]$, which we assume of common knowledge.

2.1 The game

In the second-hand market that we describe in the previous section, we assume that each agent can buy or sell and indivisible good. However, it is essential to recall the existence of incomplete information, which drives strategic behaviour from all agents to get the best possible payoff. A three-stage game describes the interaction between the store and the agent for the purchase and sale of used video games.

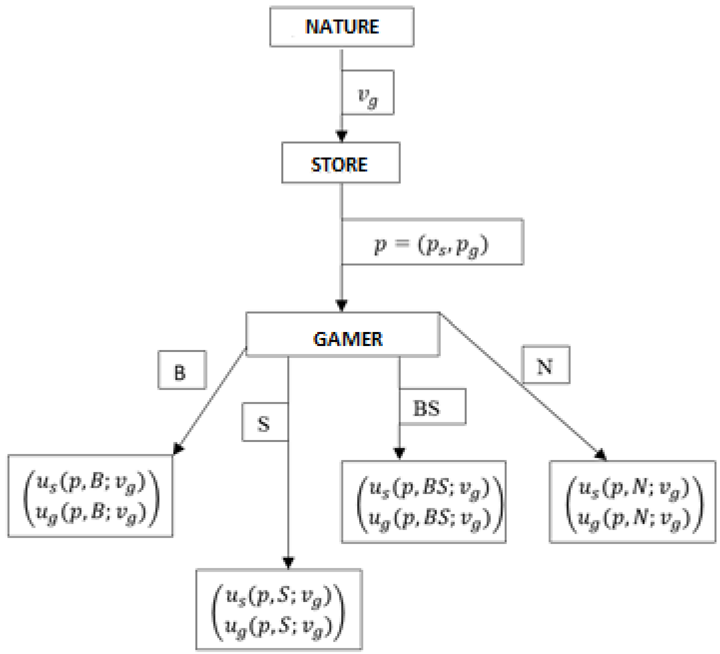

Figure 1 Extensive form of the Game.

At stage 1, nature determines the type of video player g according to the distribution f, which is observed by the video player, but not by the store. However, it is important to remark that the store knows all the games in the market. Thus, the second stage of the game starts.

During the second stage, the store sets a prices' vector $p = (p_s, p_g)$ where p_s is the price for selling good β_s, whereas p_g is the price that the store is willing to pay for good β_g. The set of all store's actions, prices vectors, is $A_s = \{(p_s, p_g) \in \mathbb{R}^2 : p_s, p_g \geq 0\}$. All agents in the market observe the prices vector p.

At the third stage, player g chooses to buy or sell (or both) a second-hand video game. Thus, player g has four actions: (i) buying β_s, (ii) selling β_g, (iii) buying β_s and selling β_g, or (iv) neither buying β_s nor selling β_g. We denote the previous actions by B, S, BS and N, respectively; so, the set of all possible gamer's actions is $A_g = \{B, S, BS, N\}$. The game finishes when the store and the player perform a transaction; each agent observes their payoff. We assume that a quasi-linear utility function represents the preferences of each agent $u_a: A_s \times A_g \to \mathbb{R}$, i.e., u_a maps an action vector (p, x) into a real number. The extensive form of the previous game is illustrated in Figure 1.

Given the initial endowment of each agent and the realisation of type v_g, the payoffs summarise the exchange, if it happens, between the store and the gamer. So, the store's payoff function is described below

$$u_s(p, x; v_g) = \begin{cases} \omega_s + p_s & si\ x = B, \\ \omega_s - p_g & si\ x = S, \\ \omega_s + p_s - p_g & si\ x = BS, \\ \omega_s & si\ x = N. \end{cases}$$

Finally, the gamer's payoff function is as follows

$$u_g(p, x; v_g) = \begin{cases} \omega_g - p_s + v_{gg} + v_{gs} & si\ x = B, \\ \omega_g + p_g & si\ x = S, \\ \omega_g - p_s + p_g + v_{gs} & si\ x = BS, \\ \omega_g + v_{gg} & si\ x = N. \end{cases}$$

2.2 The solution concept

Before introducing the concept of solution, note that the store's pure strategies match its actions because its type is unique. Then, the set of pure store strategies matches the set of prices vectors p, i.e., we have that $S_s = A_s$.

Remember, agent g knows his type and chooses an action after observing the price vector established at the end of Stage 1. This means that gamer's pure strategies are decision rules that map pairs (v_g, p) into actions in A_g; hence, a generic gamer's decision rule is a function $\sigma_g: V_g \times R \to A_g$. We denote by S_g the set of all the decision rules of agent g.

A profile of pure strategies is a vector (p, σ_g). Let $S = S_s \times S_g$ be the set of all strategies profile.

Now, note that in our three-stage game, the gamer observes the actions of the second stage while the store acts as a leader, but it does not know the type of agent. So, the solution concept that we analyse is the Sub-game Perfect Nash equilibrium. Under this solution concept, the gamer chooses the action that provides him with the highest possible payoff at the end of the third stage, and the store seeks to maximise its expected utility during the second stage by setting the prices of the indivisible goods in the market.

Definition 1. A strategies profile $(p^*, \sigma_g^*) \in S$ is a Bayesian Sub-game Perfect Nash equilibrium if and only if

a. $E\left[u_s\left(p^*, \sigma_g^*\right)\right] \geq E\left[u_s\left(p, \sigma_g^*\right)\right]$ for all $p \in A_s$, where $E[u_s]$ is the expected utility of the store, for all $p \in S_s$,

b. $u_g\left(p^*, \sigma_g^*; v_g\right) \geq u_g\left(p^*, \sigma_g; v_g\right)$ for all $\sigma_g \in S_g$ and $v_g \in V_g$.

3. Game Analysis

We proceed by backward induction to find the set of all Bayesian Subgame perfect Nash equilibria. In other words, we first compute the set of Nash equilibria of the third stage, and later the Nash equilibria of the second stage.

3.1 Player's optimal decision rule

At the beginning of the second stage, player g observes the prices of all video games in the market, i.e. he observes vector p, to choose an action in A_g. Also, player g knows his valuation vector, which means that Nash equilibria, of Stage 3, are decision rules that provide g maximises the largest possible payoff at the end of the game. The following proposition presents the Nash equilibria set of Stage 3.

Proposition 3.1 *The decision rule of g that maximises his payoff at the end of Stage 3 is*

$$\sigma_g^*(p, v_g) = \begin{cases} B & si\ v_{gs} \geq p_s, v_{gg} \geq p_g, \\ S & si\ p_s \geq v_{gs}, p_g \geq v_{gg}, \\ BS & si\ v_{gs} \geq p_s, p_g \geq v_{gg}, \\ N & si\ p_s \geq v_{gs}, v_{gg} \geq p_g. \end{cases}$$

In other words, the third stage has a unique Nash equilibrium.

Proof.

We know that player g observes its valuation vector v_g and the price vector p, while a decision rule σ_g of g is a function that maps pairs (v_g, p) into action in A_g. We need to prove that $u_g\left(\sigma_g^*(v_g, p)\right) \geq u_g\left(\sigma_g(v_g, p)\right)$ for all decision rule $\sigma_g(v_g, p)$.

Note that the price vector p is a point that divides the type space of player g into four regions, and each of them induces a decision rule that we analyse in the following cases.

Case 1. *The region determined by $v_{gs} \geq p_s, v_{gg} \geq p_g$. We analyse the following subcases*

Subcase 1.1 *Adding both inequalities, we get the expression $v_{gs} + v_{gg} \geq p_s + p_g$. Now, by adding $\omega_g$to both sides of the previous inequality, we get that $\omega_g + v_{gs} + v_{gg} \geq \omega_g + p_s + p_g$, which is equivalent to $\omega_g - p_s + v_{gs} + v_{gg} \geq \omega_g + p_g$. So, we conclude that $u_g(\sigma_g^*) = u_g(p, B; v_g) \geq u_g(p, S; v_g)$.*

Subcase 1.2 *Consider the inequality* $v_{gg} \geq p_g$; *if we add* $\omega_g + v_{gs}$ *to both sides of previous inequality, we get that* $\omega_g + v_{gs} + v_{gg} \geq \omega_g + v_{gs} + p_g$. *Now, we subtract* p_s *from the previous inequality; so, we get that* $\omega_g + v_{gs} + v_{gg} - p_s \geq \omega_g + v_{gs} + p_g - p_s$. *In other words, we have that* $u_g(\sigma_g^*) = u_g(p, B; v_g) \geq u_g(p, BS; v_g)$.

Subcase 1.3 *Note that expression* $v_{gs} \geq p_s$ *this implies that* $v_{gs} - p_s \geq 0$. *By adding* $\omega_g + v_{gg}$ *to both sides of the previous inequality, we get that* $\omega_g + v_{gg} + v_{gs} - p_s \geq \omega_g + v_{gg}$, *which implies that* $u_g(\sigma_g^*) = u_g(p, B; v_g) \geq u_g(p, N; v_g)$.

Case 2. *Now, we consider the region delimited by* $p_s \geq v_{gs}, p_g \geq v_{gg}$.

Subcase 2.1 *If we add the inequality* $v_{gs} \leq p_s$ *with the inequality* $v_{gg} \leq p_g$, *we get that* $v_{gs} + v_{gg} \leq p_s + p_g$. *Even more, the previous expression is equivalent to*$\omega_g + v_{gs} + v_{gg} \leq \omega_g + p_s + p_g$, *from which we obtain that* $\omega_g - p_s + v_{gs} + v_{gg} \leq \omega_g + p_g$. *Thus, we conclude that* $u_g(\sigma_g^*) = u_g(p, S; v_g) \geq u_g(p, B; v_g)$.

Subcase 2.2 *We add income* ω_g *in both members of inequality* $v_{gg} \leq p_g$. *So, we have that* $\omega_g + p_g \leq \omega_g + v_{gg}$; *i.e., we have that* $u_g(\sigma_g^*) = u_g(p, B; v_g) \geq u_g(p, N; v_g)$.

Subcase 2.3 *Now, consider* $v_{gs} \leq p_s$, *which is equivalent to inequality* $v_{gs} - p_s \leq 0$. *In the previous inequality, we add to both sides* $\omega_g + p_g$,*i.e., we have that* $\omega_g + p_g \leq \omega_g + p_g + v_{gs} - p_s$. *So, we conclude that* $u_g(\sigma_g^*) = u_g(p, S; v_g) \geq u_g(p, N; v_g)$.

Case 3. *The region delimited by the inequalities* $p_s \leq v_{gs}, p_g \geq v_{gg}$. *We have the following subcases.*

Subcase 3.1 *Consider the inequality* $p_g \geq v_{gg}$. *By adding* $\omega_g - p_s + v_{gs}$ *to both sides of the previous inequality, we get that* $\omega_g - p_s + v_{gs} + p_g \geq \omega_g - p_s + v_{gg} + v_{gs}$. *In other words, we get that* $u_g(\sigma_g^*) = u_g(p, BS; v_g) \geq u_g(p, B; v_g)$.

Subcase 3.2 *Now, we add* $\omega_g + p_g$ *to both sides of the inequality* $v_{gs} \geq p_s$. *Thus, we have that* $\omega_g + p_g + v_{gs} \geq \omega_g + p_g + p_s$, *which equivalent to* $\omega_g - p_s + p_g + v_{gs} \geq \omega_g + p_g$. *So, we conclude that* $u_g(\sigma_g^*) = u_g(p, BS; v_g) \geq u_g(p, S; v_g)$.

Subcase 3.3 *By adding inequalities* $v_{gs} \geq p_s$ *and* $p_g \geq v_{gg}$, *we get the expression* $v_{gs} + p_g \geq v_{gg} + p_s$. *Note that previous is equivalent to* $\omega_g + v_{gs} + p_g \geq \omega_g + v_{gg} + p_s$ *since* $\omega_g \in \mathbb{R}$ *d. Finally, we subtract from the previous expression the price* p_s, *and we get that* $\omega_g + v_{gs} + p_g - p_s \geq \omega_g + v_{gg}$. *In other words, the previous inequality implies that* $u_g(\sigma_g^*) = u_g(p, BS; vg) \geq u_g(p, N; v_g)$.

Case 4. *Now, we consider the region described by* $v_{gg} \geq p_g$ *and* $p_s \geq v_{gs}$.

Subcase 4.1 *If we add* $\omega_g + v_{gg}$ *to both sides of the inequality* $p_s \geq v_{gs}$, *we get that* $\omega_g + v_{gg} + p_s \geq \omega_g + v_{gg} + v_{gs}$. *Note that previous expression is equivalent to* $\omega_g + v_{gg} \geq \omega_g + v_{gg} - p_s + v_{gs}$. *Hence, we conclude that* $u_g(\sigma_g^*) = u_g(p, N; vg) \geq u_g(p, B; v_g)$.

Subcase 4.2 *Note that* $v_{gg} \geq p_g$ *is equivalent to* $\omega_g + v_{gg} \geq \omega_g + p_g$ *since* $\omega_g \in \mathbb{R}$. *So, we conclude that* $u_g(\sigma_g^*) = u_g(p, N; v_g) \geq u_g(p, S; v_g)$.

Subcase 4.3 *Adding inequality* $v_{gg} \geq p_g$ *to inequality* $p_s \geq v_{gs}$, *we get that* $v_{gg} + \omega_g - p_s p_s \geq p_g + v_{gs}$. *Now, we add* $\omega_g - p_s$ *to the previous expression and get that* $\omega_g + v_{gg} \geq \omega_g + v_{gs} + p_g - p_s$, *which means that* $u_g(\sigma_g^*) = u_g(p, N; vg) \geq u_g(p, BS; v_g)$.

In any case, player g gets his largest possible payoff under the decision rule σ_g^*; that is to say, we have that $u_g(\sigma_g^*(v_g,p)) \geq u_g(\sigma_g(v_g,p))$ for all decision rules σ_g. Therefore, σ_g^* is an equilibrium decision rule for player g.

□

The previous theorem illustrates the behaviour of the video player at equilibrium, which is intuitive. The player g sells his good whenever the store sets a price higher than his valuation, while he buys the good of the store when s sets a price lower than the valuation of the videos player.

The following corollary demonstrates that σ_g^* is the unique decision rule that provides him with the largest possible payoff.

It follows that the optimal decision rule in the third stage is unique.

Corollary 3.1. *Player g has a unique equilibrium decision rule at stage 3.*

Proof.

By Proposition 3.1, we know that it σ_g^* is an equilibrium decision rule for the video player since it provides the largest possible payoff when the game finishes. To prove that it is the only decision rule at equilibrium, we proceed by contradiction. So, we assume the existence of another equilibrium decision rule σ_g^{**}.

By Proposition 3.1, we know that σ_g^* gives player g the highest possible payment hen the game finishes. So, we have that

$$u_g\left(\sigma_g^*(v_g,p)\right) \geq u_g\left(\sigma_g^{**}(v_g,p)\right) \tag{1}$$

for all $(v_g,p) \in T \times S_s$.

Moreover, we assume that σ_g^{**} is another equilibrium decision rule of g at the third stage. So, σ_g^{**} provides g with the largest possible payoff when the game finishes, which implies that

$$u_g\left(\sigma_g^{**}(v_g, p)\right) \geq u_g\left(\sigma_g^{*}(v_g, p)\right). \tag{2}$$

By expressions (1) and (2), we have that

$$u_g\left(\sigma_g^{**}(v_g, p)\right) = u_g\left(\sigma_g^{*}(v_g, p)\right). \tag{3}$$

Also, we consider that payoff function u_g is quasi-linear which implies that the inverse u_g^{-1} exists. Together with expression (3), we conclude that

$$\sigma_g^{**}(v_g, p) = \sigma_g^{*}(v_g, p) \text{ for all } (v_g, p) \in T \times S_s.$$

In other words, there is a unique equilibrium decision rule for g at stage 3.

□

3.2 Equilibrium prices

By Proposition 3.1 and Corollary 3.1, there is a unique Nash equilibrium at Stage 3, namely the decision rule σ_g^*. Now, we continue with the backward induction process by computing the Nash equilibria of Stage 2 given the decision rule σ_g^*.

During the second stage, the store sets the prices of all indivisible goods (games) in the market because we consider that g has no bargaining power. However, the store does not know g's valuation vector $v_g = (v_{gg}, v_{gs})$, which is the realisation of the random vector $V_g = (V_{gg}, V_{gs})$ that follows a common knowledge probability distribution f. To simplify the analysis, we assume that V_{gg} and V_{gs} are independent and uniformly distributed random variables, but not identically distributed. Thus, we consider that V_{ga} is uniformly distributed on the interval $[m_a, n_a]$ for all $V_a \in J$. In words, player g has a minimum valuation m_a and a maximum valuation n_a about the good β_a. Consequently, the probability density function and the cumulative density function of variable V_{ga} are

$$f_{V_{ga}}(x) = \begin{cases} \dfrac{1}{n_a - m_a} & si\ x \in [m_a, n_a] \\ 0 & si\ x \notin [m_a, n_a]. \end{cases}, y\ F_{V_{ga}} = \begin{cases} 0 & si\ x < m_a, \\ \dfrac{x - m_a}{n_a - m_a} & si\ x \in [m_a, n_a), \\ 1 & si\ x \geq n_a. \end{cases} \quad (4)$$

We know that s does not know the valuation vectorv_g, i.e. it is not sure about the action that g chooses after setting the price vector p. In other words, the store maximises its expected utility function to determine the prices of all goods in the market. Thus, the expected utility of s is given by

$$E[u_s] = u_s(p, B) \Pr[B] + u_s(p, S) \Pr[B] + u_s(p, BS) \Pr[BS] + u_s(p, N) \Pr[N].$$

By substituting the payoffs that s gets at each scenario, we have that

$$E[u_s] = (\omega_S + p_s) \Pr[B] + (\omega_s - p_g) \Pr[B] + (\omega_s + p_s - p_g) \Pr[BS] + \omega_s \Pr[N]. \quad (5)$$

Proposition 3.1 establishes that g follows σ_g^* as an equilibrium strategy at stage three; that is to say, the expected utility can be rewritten as follows

$$E[u_s] = (\omega_S + p_s) \Pr[v_{gs} \geq p_s, v_{gg} \geq p_g] + (\omega_s - p_g) \Pr[v_{gs} \leq p_s, v_{gg} \leq p_g] + (\omega_s + p_s - p_g) \Pr[v_{gs} \geq p_s, v_{gg} \leq p_g] + \omega_s \Pr[v_{gs} \leq p_s, v_{gg} \geq p_g]. \quad (6)$$

Now, remember that we assume that V_{gg} and V_{gs} are independent random variables. Hence, the joint probability function f is the product between the probability functions $f_{V_{gg}}$ and $f_{V_{gs}}$. In other words, the probability of a joint event is the product of the probability of independent events.

$$E[u_s] = (\omega_S + p_s) \Pr[v_{gs} \geq p_s] \Pr[v_{gg} \geq p_g] + (\omega_s - p_g) \Pr[v_{gs} \leq p_s] \Pr[v_{gg} \leq p_g]$$
$$+ (\omega_s + p_s - p_g) \Pr[v_{gs} \geq p_s,] \Pr[v_{gg} \leq p_g] \qquad (7)$$
$$+ \omega_s \Pr[v_{gs} \leq p_s] \Pr[v_{gg} \geq p_g].$$

In expression (4), we establish that variables V_{gg} and V_{gs} follow a uniform distribution. Then, we can compute the probabilities within the expected utility function (6). Thus, we have that

$$E[u_s] = (\omega_S + p_s) \left(\frac{n_s - m_s}{n_s - m_s}\right) \left(\frac{n_g - p_g}{n_g - m_g}\right) + (\omega_s - p_g) \left(\frac{p_s - m_s}{n_s - m_s}\right) \left(\frac{p_g - m_g}{n_g - m_g}\right) + (\omega_s + p_s - p_g) \left(\frac{n_s - p_s}{n_s - m_s}\right) \left(\frac{p_g - m_g}{n_g - m_g}\right) + \omega_s \left(\frac{p_s - m_s}{n_s - m_s}\right) \left(\frac{n_g - p_g}{n_g - m_g}\right) \qquad (8)$$

The following proposition presents the equilibrium price vector of the second stage.

Proposition 3.2. *Suppose that random variables V_{gg} and V_{gs} are independent and uniformly distributed over the intervals $[m_g, n_g]$ and $[m_s, n_s]$, respectively. The store's equilibrium price vector is unique and is given by*

$$p^* = (p_s^*, p_g^*) = \left(\frac{n_s}{2}, \frac{m_g}{2}\right).$$

***Proof*.**

To compute the equilibrium price vector, we maximise the store's expected utility (see expression (8)) by following the first and second-order condition.

First-Order Condition. *To find the critical points of the expected utility function, we compute the first derivatives of $E[u_s]$.*

The first derivative of $E[u_s]$ with respect to p_s is

$$
\begin{aligned}
\frac{\partial E[u_s]}{\partial p_s} &= \frac{(n_g - p_g)(n_s - p_s + \omega_s)}{(n_s - m_s)(n_g - m_g)} + \frac{(p_g - m_g)(n_s + \omega_s - p_s - p_g)}{(n_s - m_s)(n_g - m_g)} \\
&\quad - \frac{(n_g - p_g)\,(\omega_s + p_s)}{(n_s - m_s)(n_g - m_g)} - \frac{(p_g - m_g)(\omega_s - p_g + p_s)}{(n_s - m_s)(n_g - m_g)} \\
&= \frac{(n_g - p_g)(n_s - p_s + \omega_s - \omega_s - p_s)}{(n_s - m_s)(n_g - m_g)} \\
&\quad + \frac{(p_g - m_g)(n_s - p_s + \omega_s - p_g - \omega_s + p_g - p_s)}{(n_s - m_s)(n_g - m_g)} \\
&= \frac{(n_g - p_g)(n_s - 2p_s)}{(n_s - m_s)(n_g - m_g)} + \frac{(p_g - m_g)(n_s - 2p_s)}{(n_s - m_s)(n_g - m_g)} \\
&= \frac{(n_g - p_g + p_g - m_g)(n_s - 2p_s)}{(n_s - m_s)(n_g - m_g)} \\
&= \frac{(n_g - p_g + p_g - m_g)(n_s - 2p_s)}{(n_s - m_s)(n_g - m_g)}. \qquad (9)
\end{aligned}
$$

Now, the first derivative of $E[u_s]$ with respect to p_g is

$$
\begin{aligned}
\frac{\partial E[u_s]}{\partial p_g} = &\frac{(p_s - m_s)(-p_g + \omega_s)}{(n_s - m_s)(n_g - m_g)} + \frac{(n_s - p_s)(\omega_s + p_s - p_g)}{(n_s - m_s)(n_g - m_g)} \\
&- \frac{(p_g - m_g)\,(n_s - p_s - m_s - p_s)}{(n_s - m_s)(n_g - m_g)} \\
&- \frac{(p_s - m_s)\omega_s}{(n_s - m_s)(n_g - m_g)} - \frac{(n_s - p_s)(\omega_s + p_s)}{(n_s - m_s)(n_g - m_g)}
\end{aligned}
$$

$$= \frac{(p_s - m_s)(\omega_s - p_g - \omega_s)}{(n_s - m_s)(n_g - m_g)} + \frac{(n_s - p_s)(\omega_s + p_s - p_g - \omega_s - p_s)}{(n_s - m_s)(n_g - m_g)} - \frac{(p_g - m_g)(n_s - m_s)}{(n_s - m_s)(n_g - m_g)}$$

$$= \frac{(p_s - m_s)(-p_g)}{(n_s - m_s)(n_g - m_g)} + \frac{(n_s - p_s)(-p_g)}{(n_s - m_s)(n_g - m_g)} - \frac{(p_g - m_g)(n_s - m_s)}{(n_s - m_s)(n_g - m_g)}$$

$$= \frac{(n_s - m_s)(-p_g)}{(n_s - m_s)(n_g - m_g)} - \frac{(p_g - m_g)(n_s - m_s)}{(n_s - m_s)(n_g - m_g)}$$

$$= \frac{(n_s - m_s)(m_g - 2p_g)}{(n_s - m_s)(n_g - m_g)}. \tag{10}$$

Thus, critical points of $E[u_s]$ are solutions of the following system of equations:

$$\frac{(n_g - m_g)(n_s - 2p_s)}{(n_s - m_s)(n_g - m_g)} = 0 \Rightarrow p_s^* = \frac{n_s}{2}.$$

$$\frac{(n_g - m_g)(m_g - 2p_g)}{(n_s - m_s)(n_g - m_g)} = 0 \Rightarrow p_g^* = \frac{m_g}{2}.$$

Therefore, the expected utility function of s has a unique critical point given by

$$p^* = \left(\frac{n_s}{2}, \frac{m_g}{2}\right).$$

Second-Order Condition. *Although $p^* = \left(\frac{n_s}{2}, \frac{m_g}{2}\right)$ is a critical point of $E[u_s]$, it is not clear if p maximises the expected utility function of the store. So, we need to apply the second-order condition to verify if the critical is a maximum or a minimum of the function.*

Thus, we need to evaluate the point $p^* = (p_s^*, p_g^*)$ into the Hessian of the expected utility function $E[u_s]$, i.e., we have that

$$H_{E[u_s]}(p_s^*\ ,p_g^*) = \begin{bmatrix} \dfrac{\partial^2 E[u_s\ ]}{\partial p_s \partial p_s} & \dfrac{\partial^2 E[u_s\ ]}{\partial p_g \partial p_s} \\ \dfrac{\partial^2 E[u_s\ ]}{\partial p_s \partial p_g} & \dfrac{\partial^2 E[u_s\ ]}{\partial p_g \partial p_g} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{2}{n_s-m_s} & 0 \\ 0 & -\frac{2}{n_g-m_g} \end{bmatrix}.$$

Note that $H_{E[u_s]}(p_s^*\ ,p_g^*)$ is a diagonal matrix, whose eigenvalues are the elements of the diagonal. Thus, the Hessian is a matrix with negative eigenvalues, i.e. the expected utility function of s has a maximum at $p^* = (p_s^*, p_g^*) = \left(\frac{n_s}{2}, \frac{m_g}{2}\right)$.

3.3 Agents' behaviour at equilibrium

In the previous section, we find the existence of a unique Nash equilibrium at each stage of the game described in Section 2.

Theorem 3.1 *The game described in section 2.2 has a unique Bayesian Sub-game Perfect Nash equilibrium (p^*, σ_g^*) where*

$$p^* = \left(\frac{n_s}{2}, \frac{m_g}{2}\right) \quad y \quad \sigma_g^*(p, v_g) = \begin{cases} B & si\ v_{gs} \geq p_s, v_{gg} \geq p_g, \\ S & si\ p_s \geq v_{gs}, p_g \geq v_{gg}, \\ BS & si\ v_{gs} \geq p_s, p_g \geq v_{gg}, \\ N & si\ p_s \geq v_{gs}, v_{gg} \geq p_g. \end{cases}$$

The previous Theorem establishes the agents' behaviour at equilibrium. So, the video player buys a second-hand game if the price is lower than his valuation of the store's good, while he sells his good to the store when it sets a price higher than video player's valuation of his good. Also, it is worth noticing that s sets prices in a way that $p_g^* < m_g$ and $p_s^* < n_s$. In other words, the store sets a buying price, for g's good, lower than the minimum valuation of g of his good β_g. Consequently, at equilibrium, the video player is not willing to sell his good to the store; but, it is possible that g buys the store's game since the price p_s is lower than the maximum valuation of g about β_s. Moreover, the store always sells its good if

$$\frac{n_s}{2} \leq m_s;$$

in this case, the price of β_s is lower than v_{gs} for all $v_{gs} \in V_{gs} = [m_s, n_s]$.

It is important to recall that Theorem 3.1 illustrates a unique solution for the game described in section 2.2. Hence, it is possible to perform comparative statics concerning the variation of the exogenous variables n_s and m_g.

Corollary 3.2 *The relationship between p_s^* and $n_s$is positive. The relationship between p_g^* and $m_g$is positive.*

Proof.

By Theorem 3.1, we have that $p_s^* = n_s/2$ and $p_g^* = m_g/2$. Thus, we have that

$$\frac{dp_s^*}{dn_s} = \frac{1}{2} \qquad and \qquad \frac{dp_g^*}{dn_g} = \frac{1}{2}.$$

From the previous expression, the relationship between p_s^* and p_g^* with n_s and m_g, respectively, is positive.

4. Conclusions

In this paper, we study the price formation of second-hand video games in a market where the store has market power to set price for buying and selling a good, while the video player (the consumer) has no bargaining power over such prices. Although our motivation focuses on video-games, due to the importance of second-hand markets in such industry, our model also applies to markets of durable good where there exists an agent with market power. For example, pawnshops and used car markets.

We model the interaction between a video games store and a video player as a three-stage game with private information, i.e. the store does not know the player's valuation. Also, as it is in the video games industry, the store is willing to buy any video game.

In the game's solution analysis, we proceed by backward induction. The player's equilibrium strategy is coherent with the Economic Theory since he sells/buys a good whenever the selling/buying price is lower/higher that his valuation about the goods in the market.

Concerning the results provided by the price formation mechanism, based on the heterogeneity of the goods concerning the player's preferences, we provide sufficient conditions to guarantee the existence and uniqueness of equilibrium prices. By assuming that valuations are uniformly distributed, we demonstrate that equilibrium prices are linear, and positively related to the minimum and maximum valuations of the player as we can expect. In words, the stores set a higher price for his good when the maximum valuation increases, while it sets the price of the player's price based on the minimum valuation of the player's good. So, interestingly, the store pursues not buying the player's good and always selling its good at equilibrium which formalizes the growth that second-hand videogames market lived in the early decade of 2010 (Millington, 2016).

In future works, we pretend to extend our model to the case where the video games demand induces store's valuations concerning the video games, i.e., in a game where there is private information from the store and the video player. In other words, our model ignores store's valuation. Also, questions related with the entry and exit of players and stores remain open since the model does not consider a dynamic/repeated interaction.

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